Exercise on Complex Numbers

Problem Statement

Let A, B, and C be three points with respective affixes:

$$z_A = 9 + 2i, \quad z_B = 3 - i, \quad z_C = -1 - 3i.$$

1. Determine the algebraic form of $k = \frac{z_C - z_A}{z_B - z_A}$.
2. Deduce an argument of k.
3. What can you conclude from this?

Solution

1. **Algebraic form of k**

$$z_C - z_A = (-1 - 3i) - (9 + 2i) = -10 - 5i.$$

$$z_B - z_A = (3 - i) - (9 + 2i) = -6 - 3i.$$

$$k = \frac{z_C - z_A}{z_B - z_A} = \frac{-10 - 5i}{-6 - 3i}.$$

Multiply numerator and denominator by the conjugate of $-6 - 3i$:

$$k = \frac{(-10 - 5i)(-6 + 3i)}{(-6 - 3i)(-6 + 3i)}.$$

$$(-6 - 3i)(-6 + 3i) = 36 - (-9i^2) = 36 + 9 = 45.$$

$$(-10 - 5i)(-6 + 3i) = (-10)(-6) + (-10)(3i) + (-5i)(-6) + (-5i)(3i).$$

$$= 60 - 30i + 30i - 15i^2 = 60 - 15(-1) = 60 + 15 = 75.$$

Thus:

$$k = \frac{75}{45} = \frac{5}{3}.$$

2. **Argument of k**

Since $k = \frac{5}{3}$ is a strictly positive real number, its argument is:

$$\text{Arg}(k) = 0.$$

3. **Interpretation**

The value of k shows that the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are collinear, which means that the points A, B, and C are aligned.