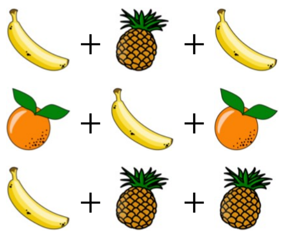
Exam 1

1. **Question**

Given the following information:



= 417

= 266

= 600

Compute:

= ?

a. 433
b. 565
c. 600
d. 261
e. 542

Solution

The information provided can be interpreted as the price for three fruit baskets with different combinations of the three fruits. This corresponds to a system of linear equations where the price of the three fruits is the vector of unknowns x:

$x_1 =$ $x_2 =$ $x_3 =$

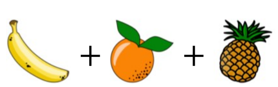
The system of linear equations is then:

$$\begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 417 \\ 266 \\ 600 \end{pmatrix}$$

This can be solved using any solution algorithm, e.g., elimination:

$$x_1 = 78,\ x_2 = 94,\ x_3 = 261.$$

Based on the three prices for the different fruits it is straightforward to compute the total price of the fourth fruit basket via:



=

$x_1 + x_2 + x_3 =$

$78 \ + \ 94 \ + 261 = 433$

a. True
b. False
c. False
d. False
e. False

2. **Question**

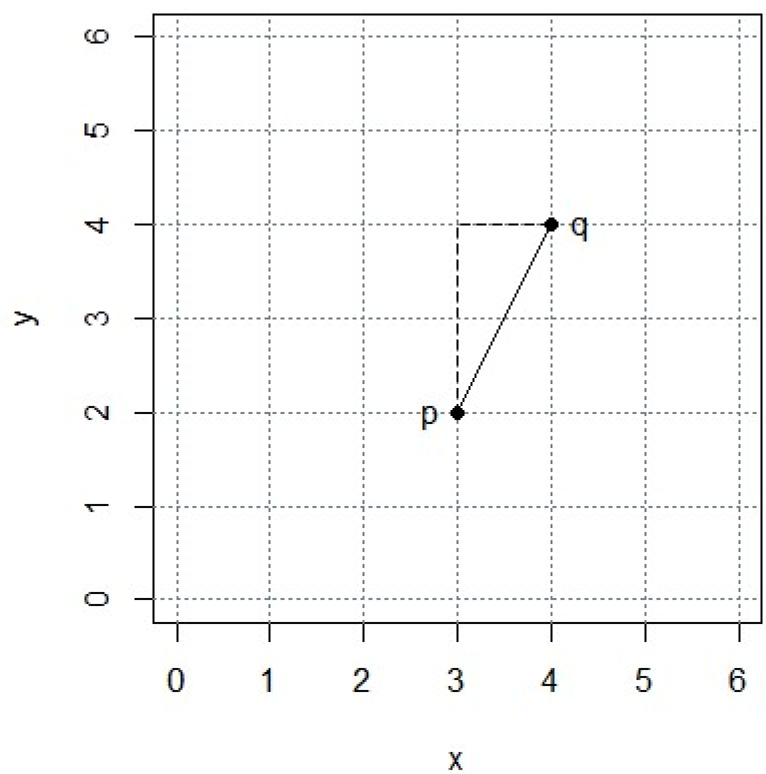
What is the distance between the two points $p = (3, 2)$ and $q = (4, 4)$ in a Cartesian coordinate system?

a. 1.139
b. 0.671
c. 1.732
d. 2.236
e. 0.237

Solution

The distance d of p and q is given by $d^2 = (p_1 - q_1)^2 + (p_2 - q_2)^2$ (Pythagorean formula).

Hence $d = \sqrt{(p_1 - q_1)^2 + (p_2 - q_2)^2} = \sqrt{(3-4)^2 + (2-4)^2} = 2.236$.



a. False
b. False
c. False
d. True
e. False

3. **Question**

What is the derivative of $f(x) = x^5 e^{3.2x}$, evaluated at $x = 0.8$?

Solution

Using the product rule for $f(x) = g(x) \cdot h(x)$, where $g(x) := x^5$ and $h(x) := e^{3.2x}$, we obtain

$$\begin{aligned} f'(x) &= [g(x) \cdot h(x)]' = g'(x) \cdot h(x) + g(x) \cdot h'(x) \\ &= 5x^{5-1} \cdot e^{3.2x} + x^5 \cdot e^{3.2x} \cdot 3.2 \\ &= e^{3.2x} \cdot (5x^4 + 3.2x^5) \\ &= e^{3.2x} \cdot x^4 \cdot (5 + 3.2x). \end{aligned}$$

Evaluated at $x = 0.8$, the answer is

$$e^{3.2 \cdot 0.8} \cdot 0.8^4 \cdot (5 + 3.2 \cdot 0.8) = 40.056741.$$

Thus, rounded to two digits we have $f'(0.8) = 40.06$.

4. **Question**

The daily expenses of summer tourists in Vienna are analyzed. A survey with 121 tourists is conducted. This shows that the tourists spend on average 136.4 EUR. The sample variance s_{n-1}^2 is equal to 148.

Determine a 95% confidence interval for the average daily expenses (in EUR) of a tourist.

a. What is the lower confidence bound?
b. What is the upper confidence bound?

Solution

The 95% confidence interval for the average expenses μ is given by:

$$\begin{aligned} & \left[\bar{y} - 1.96\sqrt{\frac{s_{n-1}^2}{n}},\ \bar{y} + 1.96\sqrt{\frac{s_{n-1}^2}{n}}\right] \\ = \quad & \left[136.4 - 1.96\sqrt{\frac{148}{121}},\ 136.4 + 1.96\sqrt{\frac{148}{121}}\right] \\ = \quad & [134.232,\ 138.568]. \end{aligned}$$

a. The lower confidence bound is 134.232.
b. The upper confidence bound is 138.568.

5. **Question**

For 58 firms the number of employees X and the amount of expenses for continuing education Y (in EUR) were recorded. The statistical summary of the data set is given by:

	Variable X	**Variable** Y
Mean	52	240
Variance	149	3259

The correlation between X and Y is equal to 0.75.

Estimate the expected amount of money spent for continuing education by a firm with 53 employees using least squares regression.

Solution

First, the regression line $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ is determined. The regression coefficients are given by:

$$\hat{\beta}_1 = r \cdot \frac{s_y}{s_x} = 0.75 \cdot \sqrt{\frac{3259}{149}} = 3.5076,$$

$$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \cdot \overline{x} = 240 - 3.5076 \cdot 52 = 57.6047.$$

The estimated amount of money spent by a firm with 53 employees is then given by:

$$\hat{y} = 57.6047 + 3.5076 \cdot 53 = 243.508.$$